According to the usual rule, if you are multiplying two measurements, the number of significant digits in the answer should be the smaller of the number of significant digits of the two measurements. We are going to examine how well this rule gives an answer whose precision reflects the accuracy of the answer.

For example, if a rectangle is measured, its width and length are $w = 23.3$ feet and $\ell = 134.7$ feet respectively, then the first measurement has three significant digits and the second has four. Therefore in calculating the area using the formula $\text{Area} = w \cdot \ell$ we should give the answer with three significant digits. Doing the multiplication on a calculator gives $23.3 \cdot 134.7 = 3138.51$ but according to the usual rule the answer should be given as $3140$ square feet or (better) $3.14 \times 10^3$ to indicate that the number of significant digits is three. Using three significant digits as we have done indicates that the actual answer is between $3.135 \times 10^3$ and $3.145 \times 10^3$. That is, it is between $3135$ and $3145$.

Looking at a second example, assume that the sides of a rectangular field are measured with the results that $w = 132.51$ feet and $\ell = 265.37$ feet.

1. If we calculate the area how many significant digits should there be in the answer according to the usual rule.

2. Give the answer with the correct number of digits according to the usual rule.

\[ \text{Area} = \underline{\hphantom{0000}} \]

3. Assuming that your answer has the correct number of significant digits what are the smallest and largest possible values for the actual area?

Smallest Area = \underline{\hphantom{0000000000}}
Largest Area = \underline{\hphantom{0000000000}}

4. What are the smallest and largest values of $w$ and $\ell$ respectively?

Smallest $w =$ \underline{\hphantom{00000000000000}}
Largest $w =$ \underline{\hphantom{00000000000000}}

Smallest $\ell =$ \underline{\hphantom{00000000000000}}
Largest $\ell =$ \underline{\hphantom{00000000000000}}

5. If the actual values of $w$ and $\ell$ are both as small as possible according to the previous problem what is the actual value of the area?

\[ \text{Area} = \underline{\hphantom{0000000000000000000000000}} \]

6. If the actual values of $w$ and $\ell$ are both as large as possible what is the actual value of the area?

\[ \text{Area} = \underline{\hphantom{0000000000000000000000000}} \]

7. The two methods used above for finding the smallest and largest possible values of the area (the method from problem 3 and the method used in problems 5 and 6) each have advantages and disadvantages. Write a paragraph describing the advantages and disadvantages of each.