Many images below are excerpts from the multimedia textbook. You can find them there and in your textbook in section 4.5.

We are now going back to solving equations. We have so far studied the solution of linear, quadratic, polynomial, rational and radical equations. Each type required its specific steps. We also learned to use substitution in various settings to help reduce the complexity of an equation.

We will now add two new types of equations to our palette: Exponential and logarithmic equations.

An equation in which a variable occurs in the exponent is an exponential equation, and an equation in which a variable occurs in the realm of a logarithmic function is a logarithmic equation.

Before we go into details let us remember two important facts about solving equations:

1. We can always solve equations graphically, no matter what type of equation we are dealing with. We just need to be sure that we have a fairly good graph of the functions involved so that we can be certain that we found all possible solutions.
   In the case of an exponential or logarithmic equation we will see that it is often necessary to solve it graphically, simply because there are no algebraic means to solve it. It will be important for us to distinguish between those equations we can solve algebraically and those we only can solve graphically.

2. Solving equations always involves "undoing" the operations of the function. When we solve quadratic equations we are taking square roots, when we solve radical equations we are squaring the equation. This means when we have equations containing exponential expressions we will use logarithms and vice versa.

The main principle to solve an exponential equation is:
This is a consequence of the exponential function being one-to-one. We can also view this principle as "applying the logarithmic function \( \log_a \) to both sides". That means:

The solution of the equation \( a^x = b \) is \( x = \log_a b \)

And similarly for the logarithmic equations

The solution of the equation \( \log_a x = b \) is \( x = a^b \)

Watch the video for section 4.5.

Work through Examples 1 and 2 in section 4.5 starting on page 333. Discussion Question 1: Explain how you solve the equations \( 2^{3x-7} = 32 \) and \( 3^x = 20 \) algebraically. What is the common principle in solving both equations?

Work through Example 5 in section 4.5 on page 337. Discussion Question 2: How do you solve the equation \( \log_3 x = -2 \)?

After we have encountered the simple cases, we can now proceed to the more complicated examples. Using the properties of logarithms we can often combine several logarithmic expressions with the same base to one expression and then apply our main principle.

Work through Example 6 in section 4.5 starting on page 337. Discussion Question 3: Explain the idea used to solve the equation \( \log x + \log(x+3) = 1 \).
Discussion Question 4: Can you use a similar idea as the one from Example 6 in solving the equations below? Explain in which cases it is possible and in which not.

a. \((2^x)(2^{2x+1}) = 16\)

b. \(\log(x+1) \log(x-2) = 3\)

c. \(2^x + 2^{2x+1} = 16\)

In some cases we can use substitution to help us out:

Work through Example 4 in section 4.5 starting on page 335.

Discussion Question 5: What was the key reason that you can use substitution in this case? Give another example similar to the problem. Does substitution work for solving \(2^x + 2^{2x+1} = 16\)?

Work through Example 8 in section 4.5 starting on page 339.

Discussion Question 6: Explain why this problem cannot be solved algebraically. Make up another problem like it.