Week 8 - Problems

1. Can you obtain the graph of \( f(x)=(1/2)^x \) from the graph of \( f(x) = 2^x \) by a transformation? How? Explain.

2. Do the interactive discovery on page 300. Answer the questions: What relationship do you see between the base \( a \) and the shape of the resulting graph of \( a^x \)? What do all the graphs have in common? How do they differ?

3. Find an exponential function obtained by horizontal and vertical shifts from a function of the form \( f(x) = a^x \) with horizontal asymptote \( y = 3 \) and a \( y \)-intercept of \( (0, 5) \). Explain how you went about solving this problem.

4. Problem 26 in section 4.1 on page 294.

5. Explain how you find \((f \circ g)(x)\) and \((g \circ f)(x)\) in Example 1b in section 4.1 on page 283. Don't "do" the example, but describe the procedure.

6. Explain how you find the domain of \((f \circ g)(x)\) and \((g \circ f)(x)\) in Example 2 in section 4.1 on page 284. Don't "do" the example, but describe the procedure.

7. Explain how one can decompose the function \( h(x) = \frac{1}{(x-2)^4} \) in two different ways as composition of two functions \( f(x) \) and \( g(x) \), that is \( h(x) = (f \circ g)(x) \).

8. Explain in your own words what \( f^{-1} \) means. Use a couple of examples to illustrate your explanations.

9. Work through Example 6 in section 4.1 on page 288. How do you prove that a function is one-to-one?

10. Work through Example 7 in section 4.1 on page 288. How do you prove that a function is not one-to-one?

11. Redo problem 41 in section 4.2 on page 308 with \( N(t)=45(1.018)^t \).
12. Find a function \( f(x) \) so that \( h(x) = \frac{x^2}{x^2 + 1} \) is equal to \( (f \circ g)(x) \) with \( g(x) = x^2 - 2 \).

13. Let \( f(x) = x^2 - 3x + 1 \) and \( g(x) = x + 1 \) find \( (f \circ g)(x) \) and \( (g \circ f)(x) \).

14. Make a hand drawn graph of \( f(x) = 3 - 2^{-x} \).

15. Make a hand drawn graph of \( f(x) = 2^{-x} - 3 \).

16. Restrict the domain of \( f(x) = (x-4)^2 \) so that the resulting function is one-to-one and find the inverse function for your new function.

17. Using your graphing calculator graph \( h(x) = (x-2)^2 \cdot 2^x \). Describe the general shape of the graph considering the graphs of the functions \( f(x) = (x-2)^2 \) and \( g(x) = 2^x \) and multiplying the corresponding y-values for a given x-value. Approximate the relative minimum and maximum values of the function \( h(x) \).

18. Problem 69 in section 4.2 on page 310.

19. Problem 104 in section 4.1 on page 297.

20. Is it always, sometimes, never true that \( f^{-1}(x) = 1/f(x) \)? Give examples.