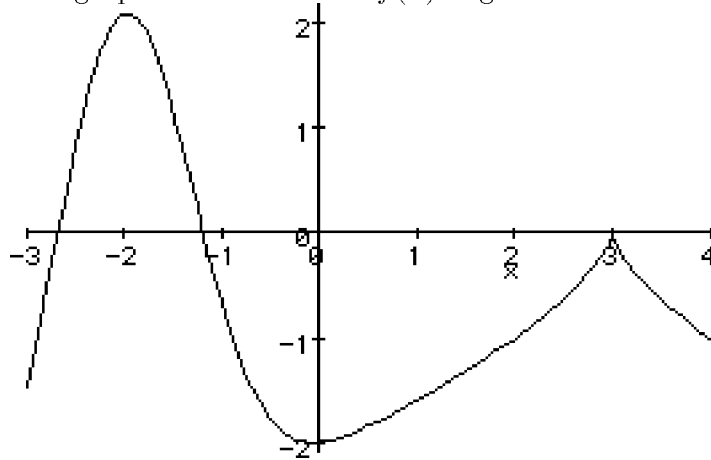


Calculus I Sample Final Exam

Name:

Instructions: Present solutions, not just final answers. Use test-taking strategy; do the ones you know and save the puzzlers till later.

1. The graph of *derivative* of $f(x)$ is given below:



(a) (4 points) Sketch the graph of $f''(x)$.

(b) (2 points) For which x -values is f increasing? decreasing?

(c) (2 points) For which x -values is f concave up? concave down?

(d) (4 points) If $f(0) = 0$, sketch the graph of f .

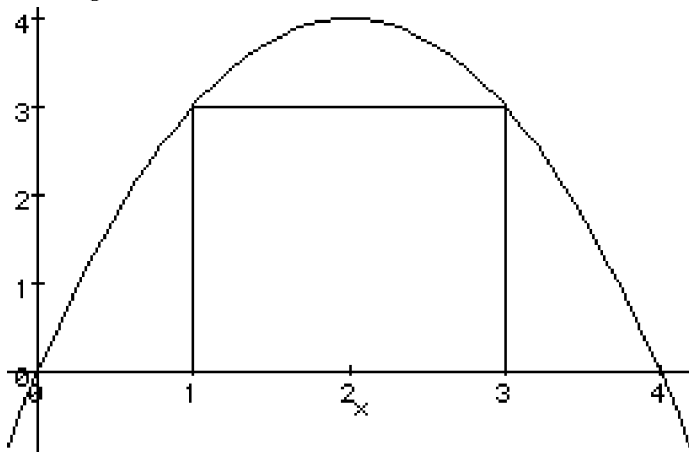
2. Evaluate the following limits. You may use L'Hopital's rule on at most *one* of the limits:

(a) (3 points) $\lim_{t \rightarrow \pi/6} \frac{\sin(3t) - 1}{t - \pi/6}$

(b) (3 points) $\lim_{h \rightarrow 0} \frac{\int_x^{x+h} f(t) dt}{h}$

(c) (3 points) $\lim_{x \rightarrow -1^+} \frac{1 - 2x^2}{x + 1}$

3. (12 points) A rectangle is positioned so that its bottom two corners are on the x -axis and its top two corners are on the graph of $y = 4x - x^2$, as diagrammed below:



If the x -coordinate of the upper right hand corner of the rectangle is increasing at the rate of 0.5 units/second, how fast is the *area* of the rectangle changing when $x = 3.5$?

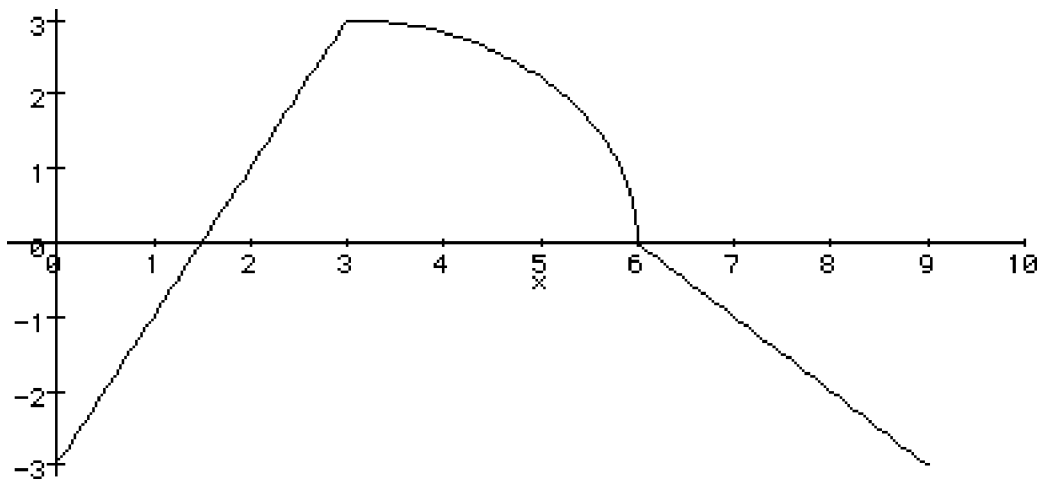
4. I'm driving my car at 90 mph (132 ft/sec), in a 45 mph zone, and I see police car lights up ahead about $\frac{1}{4}$ mile (1320 ft). I stomp on the brakes, which, even in the snow, slow down my car at the rate of 10 ft/sec².

(a) (10 points) How far will I have traveled by the time the car stops?

(b) (2 points) Do you think I will get ticketed? Why or why not?

5. The function $h(t)$ represents the height of an elevator at a given time t . The velocity of this elevator at time t is $v(t)$.
- (a) (3 points) Express the *net distance* traveled by the elevator between the times of $t = 0$ and $t = 6$ as a definite integral. Use the information provided on the graph below to evaluate this definite integral. (The curved portion is part of a circle).

- (b) (3 points) Express the *total distance* traveled by the elevator between the times of $t = 0$ and $t = 6$ as a definite integral. Use the information provided on the graph below to evaluate this definite integral. (The curved portions is part of a circle)



6. (10 points) State the Fundamental Theorem of Calculus, both parts.

7. Calculate the derivatives of the following functions:

(a) (5 points) $f(x) = \ln(2x) \arcsin(e^{3x})$

(b) (5 points) $g(x) = \int_1^x \sin(t^2) dt$

8. Calculate the following integrals:

(a) (3 points) $\int_0^1 (x^2 + e^{-x}) dx$

(b) (3 points) $\int \left(e^{\sqrt{f(x)}} \cdot \frac{1}{2\sqrt{f(x)}} \cdot f'(x) \right) dx$

(c) (3 points) $\int \left(2^x + \frac{3}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} \right) dx$

9. Consider the function

$$F(x) = \int_0^x t e^{-2t} dt.$$

(a) (3 points) Find the intervals of increase and decrease of F .

(b) (3 points) Find the intervals where F is concave up and down.

(c) (4 points) Sketch the graph of $y = F(x)$.

10. (10 points) A flower bed will be in the shape of a sector of a circle (a pie-shaped region) of radius r and vertex angle θ . Find r and θ if its area A is a constant and the perimeter is a minimum.