Abduction

No, we’re not kidnappping anyone....

Abduction

- Deduction -finding the effect, given the cause and the rule
- Induction - finding the rule, given the cause and the effect
- Abduction - finding the cause, given the rule and the effect

Abduction

- Deduction :“All men are mortal. Socrates is a man. Therefore, Socrates is mortal”
- Induction:“ All swans we see are white. Therefore all swans are white.”
- Abduction:“Drunk people do not walk straight. Jack is not walking straight. Therefore, Jack is drunk”

Abduction

- If a then b
- b
- ----------------
- a

Abduction

- You have a fever, a sore throat and a headache. It is the middle of December. You go to the doctor. After examining you, the doctor says "You have the flu".
- It appears that the doctor is reasoning as follows: Symptoms --> Disease

Abduction

- Let D= disease S=symptom
- $P(S \mid D) = P (S \cap D) / P(D)$
- $P(S \cap D) = P (D \cap S) = P(S|D)P(D)$
- $P(D|S) = P(D\cap S)/P(S) = P(S|D)P(D)/P(S)$
- We can express the probability of a disease given a symptom in terms we already have!

Abduction

Diseases

P(pickled liver) = 2^{-17}

P(iron-poor blood) = 2^{-13}

Symptoms

P(yellow skin) = 2^{-10}

P(bloodshot eyes) = 2^{-6}

Conditionals

P(yellow skin | iron-poor blood)=2^{-1}

P(yellow skin| pickled liver)=2^{-3}

P(bloodshot eyes|iron-poor blood)=2^{-6}

P(bloodshot eyes| pickled liver)=2^{-1}

Abduction

A patient has yellow skin....

P(pickled liver | yellow skin) = P(yellow skin|pickled liver)*P(pickled liver)/P(yellow skin)

$$= 2^{-3} * 2^{-17} / 2^{-10} = 2^{-10}$$

P(iron-poor blood | yellow skin) = P(yellow skin|iron-poor blood)*P(iron-poor blood)/P(yellow skin)

$$= 2^{-1} * 2^{-13} / 2^{-10} = 2^{-4}$$

Conclusion: patient has iron-poor blood

Abduction

We select the disease that maximizes the following

$$D_{MAP} = \underset{D_i}{\operatorname{argmax}}\ P(D_i|S) = \underset{D_i}{\operatorname{argmax}} P(S|D_i)P(D_i)/P(S)$$

Note 1: This is sometimes called the MAP (Maximum a Posteriori) estimate. We'll discuss further later.......

Note 2: one can disregard P(S) in the denominator, as it plays no role in selecting the estimator

Abduction

The plot thickens.......the patient has TWO symptoms(S1 and S2)....yellow skin AND blood-shot eyes.

P(D|S1 and S2)=P(D)P(S1 and S2|D)/P(S1 and S2)

Abduction

For single disease with single symptoms:
Assume m diseases and n symptoms, then
$P(D|S)=P(S|D)P(D)/P(S)$
$P(S|D)$ requires mn numbers
$P(D)$ requires m numbers
$P(S)$ requires n numbers

Total storage is $mn+m+n \sim mn$

Abduction

$P(D|S1 \text{ and } S2)=P(D)P(S1 \text{ and } S2|D)/P(S1 \text{ and } S2)$

$P(S1 \text{ and } S2|D)$ requires mn^2
$P(S1 \text{ and } S2)$ requires n^2

Total storage is $mn^2+m+n^2 \sim mn^2$
Suppose there are more symptoms??

Abduction

We make the *Naive* assumption:

a) the symptoms are independent among people at large

b)the symptoms are independent within the subset of people with disease D

Abduction

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Abduction

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$$P(S_i|S_j \text{ and } D) = P(S_i|D)$$

This permits:

$$P(S_i \text{ and } S_j|D)=P(S_i|d)P(S_j|D)^*$$

$$P(D|S_i \text{ and } S_j)=P(D)P(S_i|D)P(S_j|D)/P(S_i)P(S_j)$$

* Problem: prove this follows from b)

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$$P(D|S_i \text{ and } S_j)=P(D)P(S_i|D)P(S_j|D)/P(S_i)P(S_j)$$

We define $I(D|S) = P(S|D)/P(S)$

$$P(D|S_1 \text{ and } S_2 \text{ and } ....S_n)=P(D)I(D|S_1)I(D|S_2)....I(D|S_n)$$